Cyber-Physical System Enabled Nearby Traffic Flow Modelling for Autonomous Vehicles

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Abstract*—We propose a nearby traffic flow modelling solution based on built-in Cyber-Physical System (CPS) sensors of autonomous vehicles. Our goal is to enhance the offline route planning and driving decision adjustment based on the first-hand traffic information, especially during poor Internet connection moments. Specifically, our model helps to select the optimal speed on a road, the optimal distance for timing to brake, and the safe distance from other vehicles to keep. Moreover, our model can also assist neighboring autonomous vehicles by communicating required information through Ad-Hoc network communications or through a centralized cloud. In detail, we first focus on the unique characteristic of traffic flow (such as traffic rule, avoid collision behaviours), and then build a comprehensive model to handle multiple scenarios. Technically, our model uses density functions of velocities, the differential equation of traffic flows, and the traffic viscosity with information collected from the traffic flow, the distances between vehicles, the amount and density of vehicle, the instant velocity, the speed limit, and the momentum to analysis the the driving scene. We evaluate our model with real traffic data collected by in-vehicle CPS sensors to the proposed nearby traffic flow model. Results show that our work can accurately conduct offline estimation on nearby traffic signal influence, and reveal the correlations among velocity, density and (spatial and temporal) location to adjust route during runtime.

Keywords*—*Cyber-Physical System, Autonomous Vehicles, Traffic Flow Modeling, Intelligent Transportation, Internet of Things, Machine Learning, Big Data, Cloud Computing

I. Introduction

With the rapid development in Cyber-Physical Systems (CPS), cloud computing, machine learning, and artificial intelligence technologies, the autonomous vehicles have advanced on road. Some the well known autonomous vehicle projects are Google's Waymo project, Tesla's AutoPilot, and Uber's self-driving car. All of these autonomous vehicle projects are heavily relied on the CPSs. CPSs are tightly coupled systems of hardware and software providing large-scale, closed-loop control or management of high-level, complex dynamical systems [1].Specifically, the CPS for an autonomous vehicle consists of two levels: *Local Vehicle Client* and *Remote Cloud*, as illustrated in Fig. 1. In the *Local Vehicle Client* level, there are three main steps, including localization, perception, and vehicle control, for a car to make driving decisions and adjustments (such as speed controlling, lane switching, and distance keeping). In the *Remote Cloud Level*, the centralized datacenter offers functionalities such as traffic flow monitor, route plan and optimization, accident collection, traffic scheduling, by updating route plan based on traffic information collected.

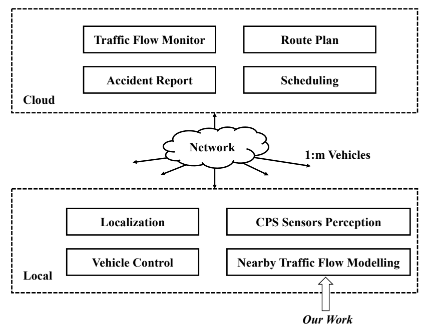


Fig. 1: Local and cloud functionalities of CPS-enabled autonomous vehicle systems.

However, in the process of making transportation intelligent and integrated, both the decision system of autonomous vehicles and big data traffic control system require the analysis of traffic flow data. Unfortunately, there is only one centralized traffic flow monitor in the cloud, and there is no local traffic flow detection module. The latter is important because (1) cloud-obtained nearby traffic information is not as updated as first-hand data, since the network delay is inevitable; and (2) the Internet connection is not always guaranteed, so completely replying on the remote cloud for updated traffic flow information is risky.

Motivated by these limitations, we propose a nearby traffic flow model based on the real-time data collected by in-vehicle CPS sensors, as the "nearby traffic flow modelling" module shown in Fig. 1. Our methodology focuses on the unique characteristic of traffic flow (such as traffic rule, and avoiding collision behaviours), and builds a comprehensive model for multiple scenarios. Technically, the proposed model is composed of velocity-density functions, traffic flow fluid dynamics partial differential equation, and traffic flow viscosity partial differential equation. To sum up, our work has the following major contributions:

- We use in-vehicle CPS sensors to enhance the offline route planning and driving decision adjustment based on first-hand traffic information, especially during poor

This work was partially supported by NSF grant CNS-1452751. The first two authors contributed equally to this work.

internet connection moments.
- Our traffic flow model can select the optimal speed on a road, the optimal distance and timing to brake, and maintain the safe distance to other vehicles to keep.
- Our model can also assist neighboring autonomous vehicles by communicating required information through Ad-Hoc network communications or through a centralized cloud.

We evaluate our model with real traffic data collected by in-vehicle CPS sensors, and results show that our solution can accurately conduct offline estimation on nearby traffic signal influence, and reveal the correlations among velocity, density and (spatial and temporal) location to adjust route during runtime.

The rest of this paper is organized as follows. Sec. II presents literature review. Sec. III formulates the problem and introduces our model. Experimental evaluation results and analysis are discussed in Sec. IV. We present the conclusions in Sec. V.

II. Related Work

Cyber-physical systems have emerged as a cutting edge technology for next-generation industrial applications, and are undergoing rapid development and inspiring numerous application domains [2]. CPS are tightly coupled systems of hardware and software providing large-scale, closed-loop control or management of high-level, complex dynamical systems. CPS-based Vehicular Ad-Hoc Networks (VANETs) [3] in particular are an active area of research. Study [4] proposed a framework for the development of predictive manufacturing CPS that includes capabilities for attaching to the Internet of Things, and capabilities for complex event processing and Big Data algorithmic analytic. [5] presented a novel CPS application for energy management framework toward autonomous electric vehicle in smart grid. [6] developed a multi-layered context-aware architecture and introduced two crucial service components, vehicular social networks and context-aware vehicular security. Furthermore, [7] proposed a new type of efficiency attack which can be used to degrade the performance of automated vehicular transportation systems.

Study [8] investigated under which circumstance the presence of a single autonomous vehicle can locally stabilize the flow, without changing the way the humans drive. A framework that utilized different models with technology-appropriate assumptions to simulate different vehicle types with distinct communication capabilities is presented in [9]. The research on potential benefits when autonomous vehicles were introduced on larger scale in the road transportation system is done by [10]. Machine learning technologies [11]–[14] and modelling algorithms [15]–[17] also play key roles in automated vehicular transportation systems. Study [18] focused on generating models for these concepts and using them to drive microscopic traffic simulations built upon real world data. Study [19] improved traffic safety and provide computational services to road users by introducing a novel cloud computing model applied to the vehicular Ad-Hoc networks. Study [20] analyzed and discussed big data solutions [21], [22] that can be leveraged to address some of the emerging challenges of autonomous vehicles cloud control system.

III. CPS-enabled Nearby Traffic Flow Modelling

Traffic flow study is essentially the topology of vehicles, which is calculated by two fundamental factors, i.e., velocity and density. Specifically, "velocity" could be the instantaneous velocity of a vehicle or could also be the average of multiple vehicles in a section of the road. "Density" is the number of vehicles in a unit length of a road section. In this paper, we focus on how to build the nearby traffic model "locally" and "offlinely" (i.e., without interacting with cloud), based on the first-hand data collected by in-vehicle CPS sensors [23].

Traditionally, vehicles are usually analogized as part of a fluid flow in the traffic flow theory [24] to simplify the calculation, In fact, both vehicles and fluid flow have common factors such as fluidity, viscosity and compressibility. However, it is hard to improve the accuracy of this analogy due to the following limitations:

- Fluid flow consists of a large amount of randomly moving particles, which are constantly colliding with each other; while the traffic flow is made of individual vehicles that follow a certain rule and attempt to avoid collision if possible.
- Lots of factors in fluid flow do not have corresponding counterparts in traffic flow, and some of the common factors are hard to measure during runtime. In practical, these factors often are ignored.
- There are more scenarios in traffic that are needed to be considered, due to road condition, vehicle behavior, and traffic rules.

Fig. 2: Example of "Local and cloud levels" of CPS-enabled autonomous vehicle systems.

Therefore, we are motivated to build a new model to address these challenges brought by the traditional fluid theory with the consideration of different scenarios, such as: continuous traffic flow scenario, vehicle-following scenario, traffic flow for non-ramp/intersection scenario, traffic pressure scenario, and viscous traffic flow scenario.

In detail, multiple types of local built-in CPS sensors that are involved in this modelling, as shown in Fig. 2. They can be organized as:

- **Visual sensors:** Using dash and rear camera, radar, laser signal, and thermographic cameras to capture videos and nearby object distances. Video images are analyzed by the machine learning algorithm [25] to get awareness about the nearby cars and humans.
- **Location sensors:** Using compass, GPS receiver, altimeter and even cellular communication module to obtain the

autonomous vehicle's temporal and spatial location, and direction it is facing.
- **Motion sensors:** Using accelerometer, odometry and gyroscope to detect the autonomous vehicle's movement direction and vehicle's velocity.

The proposed model is designed to enhance the route planning and driving decision adjustment (e.g., selecting the optimal speed on a road, the optimal distance and timing to brake, and the safe distance to other vehicles to keep) based on the first-hand traffic information, especially during poor Internet connection moments. Our model can also assists neighbor autonomous vehicles through Ad-Hoc network communication [19]) or uploading to the centralized cloud. We describe details of them in the following several subsections, and we also summarize parameters in Table I.

TABLE I: Notations.

Notation	**Meaning**
Distance (x)	Distance along the direction of vehicles traveling, with unit of m.
Time (t)	Duration of vehicle travel time, with unit of s.
Vehicle number (M)	Number of vehicles in a road section, with unit of veh.
Density (k)	Number of vehicles on a unit length of road, with unit of veh/km.
Instant velocity (u)	Instantaneous velocity of a vehicle passing a certain point, with unit of m/s.
Critical velocity (u_m)	Design velocity of the road, with unit of m/s.
Max velocity (u_f)	Maximum velocity of the road limited only by physical factors, without environmental factors, with unit of m/s. Notice that $u_f = 2u_m$.
Flow (q)	Number of vehicles passing through a certain point per unit time, with unit of veh/s, defined as $q = ku$.
Momentum (N)	Product of number of vehicles and velocity, with unit of $veh \cdot m/s$.

A. Scenario 1: Continuous Traffic Flow

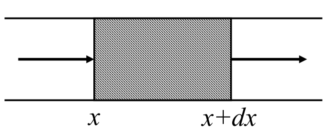


Fig. 3: Example of the traffic flow continuous function.

We first investigate the continuous traffic flow scenario. In detail, we aim to obtain the traffic flow continuous function, which is the concrete expression of the law of conservation of mass in fluid mechanics. As shown in Fig. 3, we focus on a road section ranges from x meter to $(x + dx)$ meter, and we assume that the vehicle density is k (veh), the flow is q (veh/s), the distance is $x(m)$ and time is t (s). k and q are both functions of x and t, i.e., $k(x, t)$ and $q(x, t)$.

$$\frac{\partial k}{\partial t}dx + \frac{\partial q}{\partial x}dx = \lambda = \frac{dr}{dx} \tag{1}$$

where λ is the flow arrival rate, which is a function of influx r, i.e., $\lambda = \frac{\partial r}{\partial x}$. In fact, there are two cases:

Case 1: if there is no interchanges (i.e., on/off ramps) in this road section, we have $\lambda = 0$. In other words, the in-flow and out-flow can be considered as equivalent. Let the out-flow rate be $\frac{\partial q}{\partial x}dx$, and the in-flow rate to be $\frac{\partial k}{\partial t}dx$, i.e., $\frac{\partial k}{\partial t}dx + \frac{\partial q}{\partial x}dx = 0$.

Case 2: if this road section has an on/off ramp, we have $\lambda \neq 0$. Furthermore, when vehicles leave the road section, we have $\lambda < 0$ and $r < 0$; when vehicles enter the road section, we have $\lambda > 0$ and $r > 0$; and finally when there is no vehicle movement, we have $\lambda = 0$ and $r = 0$.

B. Scenario 2: Vehicle-following Traffic Flow

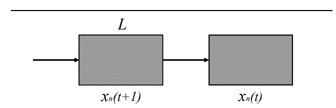


Fig. 4: Example of the vehicle-following scenario.

We next investigate the velocity-density function for the vehicle-following scenario, which is the basis of the traffic flow model. This function focuses on the trajectory of each moving vehicle in the road. We first formulate the vehicle-following scenario. As depicted in Fig. 4, a "head" vehicle with a trajectory function as $x_1(t)$ is selected. We hereby only focus on the movements of this vehicle and all back vehicles. We notate the trajectory functions of the n-th and $(n+1)$-th vehicles as $x_n(t)$ and $x_{n+1}(t)$. Then the distance between two vehicles, n and $n+1$, can be calculated as $X_n(t) = x_n(t) - x_{n+1}(t)$. To simplify the problem, we assume each vehicle has a similar length (L) and there is no "overtakings" between vehicles, then we have $L \leq X_n(t) < +\infty$. Obviously, the velocity of the behind vehicle of two consecutive vehicles is negatively influenced by the distance between that vehicle to the frond vehicle, and assume this relationship is $u_{n+1}(X_n)$. Additionally, when the velocity of $(n+1)$-th vehicle (i.e., $u_{n+1}(t)$) satisfies the minimum distance between two vehicles (i.e., $X_n(t) = L$), the $(n+1)$-th vehicle's speed is set to its minimum speed limit, i.e., $u_{n+1}(t) \to 0$; similarly, when $X_n(t) \to +\infty$, $u_{n+1}(t)$ will strive to reach its maximum speed limit, i.e., u_m.

Next, for a random road section, assume the n+1-th vehicle is located at $< x, t >$ (i.e., location x meter at time t). Since the selection of this vehicle is random, we can use its velocity as an estimation of the average of this road section, i.e., $u(x, t)$. Assume the distance between the rear of vehicle n and the rear of vehicle $n+1$ is L_n, then we have $u(x, t) = u_{n+1}(X_n)$ for $0 \leq u \leq u_m$. Similarly, the density of this section of the road $k(x, t)$ can be estimated by the distance between $(n+1)$-th vehicle and its front (n-th) vehicle, i.e., $k(x, t) = \frac{1}{L_n}$ for $0 \leq k \leq k_m$, where $k_m = \frac{1}{L}$ is the maximum density. Additionally, for a more general form, we have $L_n = L + X_n(t)$. As a result, u is inversely proportional to K, and satisfies $u(k_m) = 0$ and $u(0) = u_m$.

We also notice that in reality, the relationship between vehicles in traffic flow is not always one-to-one, and the velocity and density follows the *Greenshields* velocity-density linear relation, i.e.,

$$u = u_m[1 - (\frac{k}{k_m})^n] \tag{2}$$

where n is a variable parameter, and in this paper, we let n represent the time sequence. By using the total variation diminishing (TVD) solution [26] to iterate Eq. 1 and Eq. 2, we have:

$$k_j^{n+1} = k_j^n - \frac{\Delta t}{\Delta x}(q_{j+\frac{1}{2}}^n - q_{j-\frac{1}{2}}^n) \tag{3}$$

where the nearby traffic flow is:

$$q_{j+\frac{1}{2}}^n = \frac{1}{2}[q(k_{j+1}^n) + q(k_j^n) - u_{j+\frac{1}{2}}^n(k_{j+1}^n - k_j^n)] \tag{4}$$

and the velocity is:

$$q_{j-\frac{1}{2}}^n = \frac{1}{2}[q(k_j^n) + q(k_{j-1}^n) - u_{j-\frac{1}{2}}^n(k_j^n - k_{j-1}^n)] \tag{5}$$

$$u_{j+\frac{1}{2}}^n = \begin{cases} \left|\dfrac{q(k_{j+1}^n) - q(k_j^n)}{k_{j+1}^n - k_j^n}\right|, & k_{j+1}^n - k_j^n \neq 0 \\ q(k_j^n), & k_{j+1}^n - k_j^n = 0 \end{cases} \tag{6}$$

$$u_{j-\frac{1}{2}}^n = \begin{cases} \left|\dfrac{q(k_j^n) - q(k_{j-1}^n)}{k_j^n - k_{j-1}^n}\right|, & k_j^n - k_{j-1}^n \neq 0 \\ q(k_j^n), & k_j^n - k_{j-1}^n = 0 \end{cases} \tag{7}$$

where j is the differential element unit.

C. Scenario 3: Non-Ramp/Intersection Traffic Flow

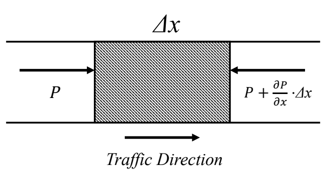


Fig. 5: Example of differential equation of traffic flow.

Although scenario 1 covers the non-ramp/intersection case, traffic flow continues function may not be accurate enough. To enhance the model for the scenario that a road section has ramps and intersections, we adopt methodologies from the compressible continuous flow theory. This is because that moving vehicles and compressible continuous flow have common properties such as velocity reduction, density increasing, and vehicle distance reduction. As illustrated in Fig. 5, the length of a section in a single lane is Δx meters, and the number vehicles in this section is $k \cdot \Delta x$. Let the lift-hand-side "force" exerting on the cross section be P. Since that the number of vehicles in this system conserves, the right-hand-side "force" can be calculated as $P + \frac{\partial P}{\partial x} \cdot \Delta x$. According to Newton's second law, we have:

$$P - (P + \frac{\partial P}{\partial x} \cdot \Delta x) = k \cdot \Delta x \cdot a \tag{8}$$

As $u(x(t), x)$ is a function of x and t, we have $a = \frac{du}{dt} = \frac{\partial u}{\partial t} + u \cdot \frac{\partial u}{\partial x}$. Apply this result to Eq. 1, we get:

$$\frac{\partial u}{\partial t} + u \cdot \frac{\partial u}{\partial x} + \frac{1}{k} \cdot \frac{\partial P}{\partial x} = 0 \tag{9}$$

This result for a road section without influx and out flux is also named as "the first differential equation of traffic flow".

D. Scenario 4: Traffic Pressure

Traffic pressure is also an important topic of autonomous vehicles route planning and driving decision adjustment. In scenario 3, we treat the traffic flow as a compressible continuous flow. In fact, the "traffic pressure" can be further defined as the particular "force", which is exerted on the traffic flow, i.e., $P(veh \cdot m/s^2)$. Physically, it is the force to push M vehicles with the acceleration of a, and this force depends on factors, such as road and vehicle conditions, and the resistance from road to vehicles. In this paper, we focus on the traffic flow under a constant situation, i.e., when $\frac{\partial k}{\partial t}$ and $\frac{\partial u}{\partial t} = 0$. Derived from Eq. 9, we have:

$$u \cdot \frac{\partial u}{\partial x} + \frac{1}{k} \cdot \frac{\partial P}{\partial x} = 0 \tag{10}$$

After differential transforming and integrating Eq. 10, we have:

$$P = P_1 + q(u_1 - u) \tag{11}$$

Notice that $q = ku$ is a constant, and P_1 and u_1 are given force and velocity.

E. Scenario 5: Viscous Traffic Flow

The last scenario is based on the fact that vehicles do affect each other in the real traffic flow. More precisely, when traffic flow reaches a certain density, vehicles will show some "viscosity characteristics" due to the cross-vehicle interference. Specifically, assume the viscosity resistance is $f_w = -2 \cdot \frac{u_1 - u}{k} \cdot \frac{\partial q}{\partial x}$, then when $u < u_1$, we have $f_w = 0$; and when $u \geq u_1$, from Eq. 9 we obtain:

$$\frac{\partial u}{\partial t} + u \cdot \frac{\partial u}{\partial x} + \frac{1}{k} \cdot \frac{\partial P}{\partial x} + f_w = 0 \tag{12}$$

If we further take derivative of Eq. 11 by using Eq. 1, we have:

$$\frac{\partial u}{\partial t} \pm \frac{u_1 - u}{k} \cdot \frac{\partial k}{\partial t} = 0 \tag{13}$$

When $u \geq u_1$, Eq. 13 takes "-". By choosing the integration interval in the range of $[u_2, u]$, we have:

$$u = u_1 + (u_2 - u_1) \cdot \frac{k_2}{k} \tag{14}$$

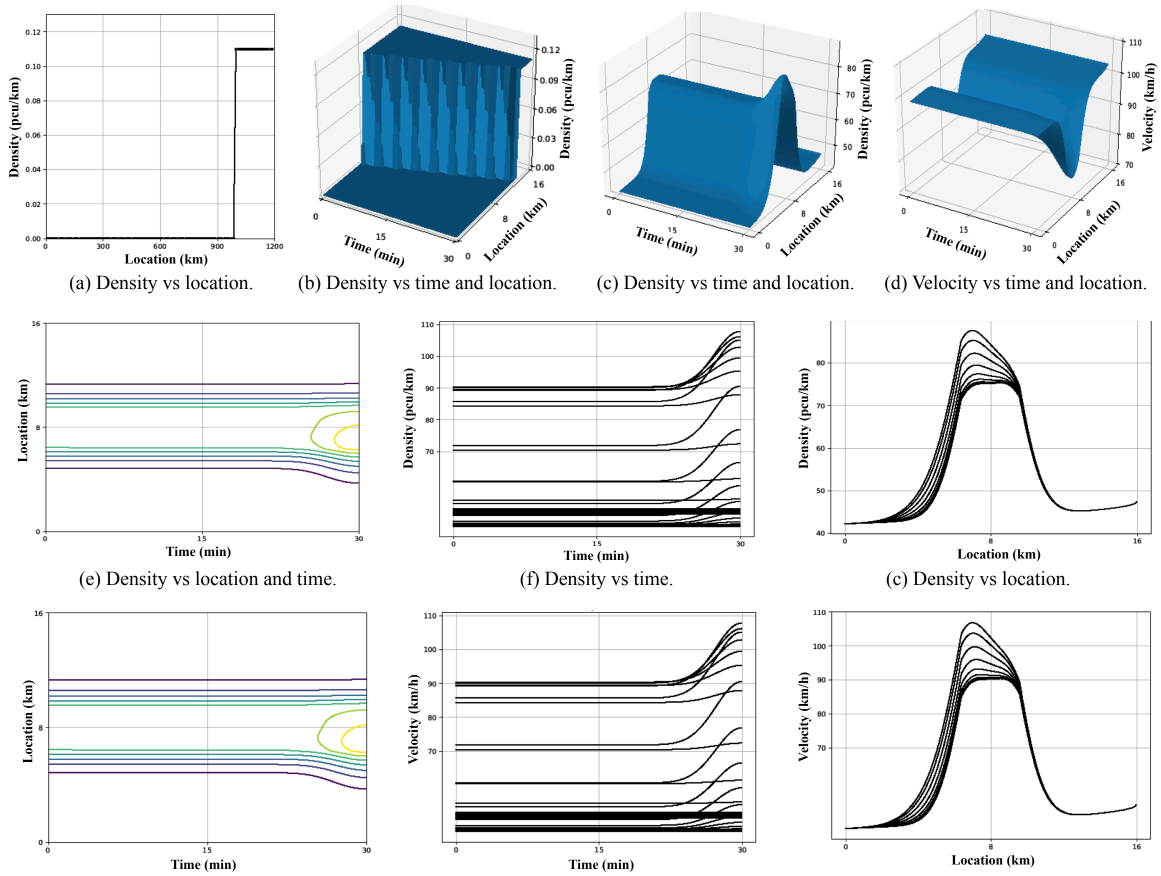
When the flow represents congestion, i.e., $u_1 = 0$, we have $ku = k_2 \cdot u_2$. This equation is called "velocity-density constant flow model". Similarly, when $u < u_1$, Eq. 13 takes "+". We have $u = u_m(1 - \frac{k}{k_j})$, which is called "Green Shield velocity-density linear relationship model".

IV. Evaluation

In this section, we feed real traffic data collected by in-vehicle CPS sensors to the proposed nearby traffic flow model. Based on the calculation results, we evaluate the traffic signal influence, and the estimation accuracy in terms of the correlations among velocity, density and (spatial and temporal) location.

A. Traffic Signal Influence

We first investigate the traffic signal's influence on the vehicle density in a certain road section. We collect data from CPS sensors of an autonomous vehicle within a 1200-meter road section. There is a traffic signal in the end of this road section. Measurement results show that the velocity in this section is about $u_f = 16.67m/s(60km/h)$, the density in the



(a) Density vs location. (b) Density vs time and location. (c) Density vs time and location. (d) Velocity vs time and location.

(e) Density vs location and time. (f) Density vs time. (c) Density vs location.

(h) Velocity vs location and time. (i) Velocity vs time. (j) Velocity vs location.

Fig. 6: Experimental results for of evaluations. (a)-(b) are for traffic signal influence. (c)-(j) are for traffic flow estimation.

TABLE II: Estimated velocity and density of highway.

Subsec.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Density (pcu/km)	52.2	64.6	67.7	70.4	74.4	74.7	75.8	731	68.2	65.3	60.5	55.9	53.4	51.1	51.9	49.8
Velocity (km/h)	98.5	93.1	87.4	84.7	82.9	82.6	80.3	83.1	90.3	92.2	94.7	96.9	99.3	102.1	105.2	106.1

end of this section is about $k_j = 0.11veh/m$ on average, and the initial density of this section is $k = 0.04veh/m$. We also have $\Delta x = 4m$, $\Delta t = 0.2s$, $J = 300$, the initial condition as $k_j^0 = 0.04$, where the differential element unit $0 \leq j \leq J$, and the boundary condition is $k_0^n = 0.04$, $k_J^n = 0.11$.

By using the proposed nearby traffic flow model, we plot Fig. 6(a) for the correlation of density and location, and Fig. 6(b) for the correlation of density time, and location. From these two figures, we observe that the density change is not a gradient process, instead, it keeps in a relatively flat range in the beginning, and suddenly rises rapidly in a very short section (around at location of $984m$ in Fig. 6(a)). This indicates that majority of vehicles start to *brake* around $984m$ away from the red light signal, instead of *slow down smoothly* when approaching the red light signal (i.e., an optimal solution). Also, we see that on average, the queue is formed at about $200m$ away from traffic signal. This information is hard to obtain online from the cloud during runtime, and it is helpful for the autonomous vehicle to better control the speed (or even change the route) according to the nearby traffic flow change caused by traffic signals, in order to save energy and to improve the ride experience.

B. *Nearby Traffic Flow Estimation Accuracy*

In this subsection, we focus on estimating accuracy of the highway scenario made by in-vehicle CPS sensors on a high-speed moving autonomous vehicle. Specifically, the tested highway section length is $8km$, and the designed velocity for this highway is $120km/h$. We divide the section into 16 subsections, i.e., $\Delta x = 0.5km$. Table II shows the traffic density and velocity results measured by the moving autonomous vehicle. Additional, the jam density calculated by the autonomous vehicles is $k_j^a = 106.7pcu/km$.

Figs. 6(c)-(d) illustrates the relationship between density, velocity, time and location and Figs. 6(e)-(f) and (h)-(j) show the aerial views of density vs location and time, and velocity vs location and time, respectively. From these figures, we observe that, after a stable period, the median value of traffic density is around $64.5pcu/km$, and the median value of traffic velocity is around $93.7km/h$. This result can enhance the autonomous vehicle's adaptive cruise control feature. Moreover, we observe that high-density waves cannot transfer forward nor backward, which validates the accuracy of our CPS-enabled nearby traffic flow estimation. This is because in our test, the in-flow and out-flow of the all these subsections are equivalent.

V. Conclusion

We present a nearby traffic flow model based on fluid theory. The proposed model is based on various information collected by CPS systems on the autonomous vehicles. Our goal is to help autonomous vehicles to make decisions and correct the wrong decisions, such as the optimal speed on a road, the optimal distance for timing to brake, and to keep a safe distance with other vehicles. Technically, our model uses local CPS sensors in the autonomous vehicle to estimate density functions of velocities, the differential equation of traffic flows, the traffic viscosity with information collected from the traffic flow, the distances between vehicles, the amount and density of vehicle, the instant velocity, the speed limit, and the momentum to analysis the driving scene.

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